Heavy Duty Platonism*

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Abstract

Heavy duty platonism (HDP) is of great dialectical importance in the philosophy of mathematics. It is the view that physical magnitudes, such as mass and temperature, are cases of physical objects being related to numbers. To my knowledge, no philosophers have openly defended HDP. However, many have assumed that it is false as a crucial premise in important arguments. These conclusions are justified only if HDP is shown to be untenable, but arguments to that effect are surprisingly hard to find. Most are content just to assert that HDP is false, while some go only as far as to point to counterintuitive implications of the view. In this paper, I organise these intuitions into five arguments against HDP and show that they each fail. I thereby establish two related truths: HDP has been unfairly ignored in the literature, and the arguments mentioned above that take the falsity of HDP as a key premise should be re-assessed.

1. Introduction

Heavy duty platonism (HDP) is of great dialectical importance in the philosophy of mathematics. It is the view that physical magnitudes, such as mass and temperature, are cases of physical objects being related to numbers. To my knowledge, no philosophers have openly defended HDP. [1] However, many have assumed that

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[1] Pincock's 2015 account of mathematical explanation in science sounds very much like HDP. Melia (e.g. 1995: 228-229) assumes that taking mathematics to play an explanatory role in science is tantamount to endorsing HDP. If he's right, proponents of the indispensability argument (e.g. Colyvan 2001) are committed to HDP.

it is false as a crucial premise in important arguments. For example: Hartry Field (1989: 186-200) rejects one theory of space-time in favour of another because it apparently implies HDP; Joseph Melia's (1995: 229, 2000: 473-474) attack on the Quine-Putnam indispensability argument rests on the assumption that HDP is implausible; Churchland (1979: 105) rejects the view that intentional states are relations between thinkers and propositions because it is supposedly analogous to HDP; and Tim Crane, who denies this analogy, agrees that HDP cannot be true (1990: 227).

These conclusions are justified only if HDP is shown to be untenable, but arguments to that effect are surprisingly hard to find in the literature. Most are content just to assert that HDP is false, while some go only as far as to point to counter-intuitive implications of the view. In this paper, I organise these intuitions into five arguments against HDP and show that they each fail. I thereby establish two related truths: HDP has been unfairly ignored in the literature, and the arguments mentioned above, that take the falsity of HDP as a key premise, should be re-assessed.

In §2, I formulate one argument based on the Lewisian distinction between intrinsic and extrinsic properties. The argument is that HDP wrongly categorises all physical magnitude properties as extrinsic. I show that a plausible and popular analysis of intrinsicality implies that HDP's physical magnitudes are intrinsic. I then consider the reply that a better analysis would characterise HDP's physical magnitudes as extrinsic. In response, I provide evidence that our intuitions about which physical magnitudes are intrinsic are misleading, so HDP need not honour them.

In §3, I formulate two arguments from Crane's discussion of HDP. The first is that the proponent of HDP must accept one of two untenable theses: that there is a metaphysically privileged measurement scale; or that an object's physical magnitude consists in its being related to all the numbers the magnitude property is measurable with. I show that both are defensible. The second argument is that HDP entails that physical objects have some of their causal powers by being related to non-causal objects, which is incoherent. I show that this is coherent and outline a theory of explanation that shows why.

In §4, I undermine two arguments offered by Field, using arguments outlined in §§2-3, at which time I will have successfully defended HDP from all objections alluded to in the literature, and so vindicated the above conclusions. However, before all this, more needs to be said about what HDP amounts to.

The most direct discussion of HDP is provided by Field (1989: 186-189). He says that platonists believe there are 'relations of physical magnitude that relate physical things and numbers' (1989: 186). For example, a 10kg bag of sand bears the *mass in kilograms* relation to the number 10. Call these 'platonic relations'. What separates different kinds of platonism is what they tell us about these relations: weaker forms tell us they are derivative of more fundamental properties or relations that hold of physical objects alone, while HDP says these relations are fundamental and 'not explainable in other terms' (1989: 186). (Field perhaps took 'not explainable in other terms' and 'fundamental' to be synonymous; I don't want to commit to this.)

Another way of describing the difference is as follows. Weaker forms of platonism imply that there is a purely physical fact about a 10kg bag of sand that makes it the case that it bears the mass in kilograms relation to the number 10, while HDP implies that there isn't.

HDP allows more fundamental properties in virtue of which a given physical magnitude holds. For example, the heavy duty platonist can explain the mass relation between a brick and the number 10 in terms of the mass relations between the particles composing the brick and the relevant numbers, in which case the relation between the brick and the number 10 would not be absolutely fundamental, but fundamental relative to the brick. HDP does not allow that the mass of the brick is explained by a property instantiated by the brick alone. Relative to the brick, HDP has it that the relations it stands in to numbers are the fundamental facts concerning its physical magnitudes.

2. Arguments from Lewis

Here I formulate one argument against HDP based on the Lewisian distinction between intrinsic and extrinsic properties. These notions are typically introduced by stating various platitudes about each. Here is a list from Lewis:

> A sentence or statement or proposition that ascribes intrinsic properties to something is entirely about that thing; whereas an ascription of extrinsic properties to something is not entirely about that thing, though it may well be about some larger whole which includes that thing as part. A thing has its intrinsic properties in virtue of the way that thing itself, and nothing else, is. Not so for extrinsic properties, though a thing may well have these in virtue of the way some larger whole is. The intrinsic properties of something depend only on that thing; whereas the extrinsic properties of something may depend, wholly or partly, on something else. If something has an intrinsic

> property, then so does any perfect duplicate of that thing; whereas duplicates situated in different surroundings will differ in their extrinsic properties. (1983: 111-112)

There are generally thought to be clear-cut examples of each. The property being a stone is thought to be intrinsic, for something's being a stone does not appear to involve anything distinct from itself. *Being 20 miles away from a pig*, however, is extrinsic: it involves something other than its bearer—a pig.

Philosophers have claimed that some physical magnitude properties are clear-cut cases of intrinsic properties (cf. Stalnaker 1987: 9; Crane 1990: 227; Mumford 2006: 471-480; Molnar 2003: 131-137, and Ellis 2001: 114-115.) Intuitively, an object's mass involves only that object, but HDP implies that an object has its mass by being related to a number. So, HDP implies mass is extrinsic. The argument we might formulate from these intuitions runs as follows:

The intrinsic argument

I1: Some physical magnitudes are intrinsic properties.

I2: According to HDP, all physical magnitudes are extrinsic properties.

I3: HDP is false.

I will present three responses to the intrinsic argument. The first is not satisfying, but helps clarify the argument and highlight important features of HDP. I will discuss this response first. The second and third responses involve rejecting I2 and I1, respectively, and are more satisfying.

I2 says that HDP's physical magnitudes are extrinsic properties. This is misleading: they are not properties at all, but *relations*. Granted, associated with each relation are *relational properties* that hold of each relatum. For instance, the relation holding between x and y just in case x is taller than y has the associated relational properties *being taller than* y, which holds of x alone, and *being shorter than* x, which holds of y alone. The relation is not identical with these properties, but it is intimately related: the properties are instantiated just in case the relation obtains. Intuitively, the relational properties hold in virtue of the relation's obtaining, rather than the other way round. The relation is more fundamental.

The same goes for HDP's physical magnitudes: if o is related to 10 by the mass in kilograms relation, there is a relational property that holds of o alone: *bearing the mass in kilograms relation to* 10. But this property holds in virtue of the relation's obtaining. Moreover, the relation appears only to involve the physical object and the number, and nothing else. Far from implying that physical magnitudes are extrinsic relations, HDP implies they are *intrinsic relations*, where the notion of an intrinsic relation is a straightforward generalisation of the notion of an intrinsic property, and is introduced in a similar manner:

> An n-place intrinsic relation is an n-place relation that n things stand in virtue of how they are and how they are related to each other, as opposed to how they are related to things outside of them and how things outside of them are. (Weatherson and Marshall 2013: §1.3)

Hence, I2 is false, and the intrinsic argument is unsound. This response attacks the letter, rather than the spirit, of the argument. A more charitable interpretation has the argument concerned with the properties that HDP attributes to physical objects alone. Say o has mass 10kg. Intuitively, this partly involves o's instantiating an intrinsic property. According to HDP, however, the only thing this implies about o alone is that it instantiates the relational property of bearing the mass in kilograms relation to 10, which looks extrinsic. So HDP does mischaracterise the properties of the physical objects.

This is too quick. The property of having bigger quadriceps than biceps is a property that most people instantiate. It is relational: for it to be instantiated by Jill, the *bigger than* relation must hold between Jill's quadriceps and her biceps. Nevertheless, there is still some sense in which this property involves only Jill, so it is intuitively intrinsic. It is clear that relational properties are not automatically extrinsic (cf. Weatherson and Marshall 2013: §2.1).

Further, some relational properties are instantiated because a relation holds between an object and some necessary state of affairs. For instance, Bill instantiates the property of being such that Jill is alive or not alive and the property of being such that $2 + 2 = 4$. One might think that such properties are extrinsic because they appear to depend on states of affairs distinct from the bearer. However, one might also think that they are intrinsic because objects have these properties no matter how the rest of the world turns out. Because it is necessary that Jill is alive or not alive and that $2 + 2 = 4$, Bill is necessarily such that Jill is alive or not alive and such that $2 + 2 = 4$, so Bill's having these properties does not depend on how things go with other objects. Our intuitions can lead either way.

When a theoretical distinction faces problematic cases, it must be made more precise. We need an analysis of the intrinsic/extrinsic distinction that pins down what it is for a property to only involve its bearer. Such an analysis should be plausible, decisive with respect to the problematic cases, and track our intuitions about the unproblematic cases.

Rae Langton and Lewis provide a plausible and popular analysis: 'a property is *intrinsic*... iff whenever two things (actual or possible) are duplicates, either both of them have the property or both of them lack it' (1998: 337). Call this the 'Lewisian analysis'. This is a plausible account of what it means to say a property involves only its bearer and it tracks our intuitions concerning the unproblematic cases. Moreover, the analysis is decisive with respect to the aforementioned problematic cases. In all worlds where there is a perfect duplicate of Bill, Jill is alive or not alive and $2 + 2 = 4$, so in all those worlds Bill will be such that Jill is alive or not alive such that $2 + 2 = 4$. According to the Lewisian analysis, these properties are intrinsic.

The same goes for HDP's relational properties. The standard assumption is that, if numbers exist, they exist necessarily. On this view, HDP implies that o will instantiate the property of bearing the mass in kilograms relation to 10 in all worlds in which there is a perfect duplicate of o. Even if we reject the standard assumption, we cannot claim there is a numberless world containing a perfect duplicate of o without assuming that the number 10 does not help make it the case that o has mass 10kg. This would beg the question. According to the Lewisian analysis, HDPs relational properties are intrinsic.

This is the second response to the intrinsic argument. Our intuitions about the intrinsic/extrinsic distinction are not robust enough to classify certain cases, so we need an analysis to make it more precise. According to a plausible and popular analysis, HDPs relational properties are intrinsic. This is a principled reason for rejecting I2 and the intrinsic argument.

Some may have a strong intuition that HDP's relational properties are extrinsic, and take the fact that the Lewisian analysis classifies them otherwise to be a good reason to reject the analysis. This brings me to the third means of defending HDP from the intrinsic argument.

Suppose that the best available analysis of the intrinsic/extrinsic distinction implies that any property involving something other than the bearer, even if that something is a number, is extrinsic. On this analysis, HDP clashes with the intuition that physical magnitude properties are intrinsic. However, if it can be shown that these intuitions are unreliable, then HDP's failure to honour them will not count against it.

Mass is often taken by philosophers to be a clear-cut case of an intrinsic property (cf. Mumford 2006: 471-480, Molnar 2003: 131-137, and Ellis 2001: 114-115). However, as William A. Bauer (2011: 89-93) argues, contemporary science appears to falsify this intuition. According to our best theory of fundamental particles, mass is extrinsic.

The Standard Model is our best physical theory of fundamental particles. The predicted discovery of the W+, W- and Z bosons in 1983 at CERN, and more recently that of the Higgs Boson in 2013, has given the theory substantial empirical support. The Standard Model tells us that the mass of a particle is not just a result of the way that particle is, but also of the particles interaction with a certain scalar field:

> [The mechanism by which a particle gains its mass] is based on the assumption of the existence of a scalar field, the "Higgs Field", which permeates all of space. By coupling with this field a massless particle acquires a certain amount of potential energy and, hence, according to

> the mass-energy relation, a certain mass. The stronger the coupling, the more massive the particle. (Jammer 2000: 162-163)

The mass of particles depends not only on the properties of the particles, but also on the properties of the Higgs Field. The Higgs Field is entirely distinct from any particle interacting with it. For any particle p, there is a world in which p does not exist while the Higgs Field does. Science tells us that the mass of a particle depends on the properties of something distinct from the particle, so, on the present view, mass is extrinsic. According to HDP, then, mass is a three-place relation holding between an object, a number, and the Higgs Field.

It is not surprising that our intuitions about which physical properties are intrinsic can go awry, since they appear to be based on naïve observation. We do not observe anything distinct on which an object's mass depends, so we take it to be intrinsic. Our judgements about which properties are intrinsic should instead be informed by our theories about their nature. Physics reveals that our initial judgements were wrong and that mass is extrinsic, and there's no reason why metaphysics can't be informative in this way, too. For example, Ted Sider (2003) shows that many go-to examples of intrinsic properties, such as being a stone, are in fact extrinsic. Whether or not something is a stone partly depends on whether it is part of a larger stone or not, so it depends on something distinct from itself. Along with the scientific example above, this provides a strong reason for rejecting arguments or theories that appeal to the intuitive intrinsicality of everyday properties (cf. Weatherson and Marshall 2013: §1.1).

HDP is a theory about the nature of physical magnitude properties. If correct, it should inform our judgements about which side of the intrinsic/extrinsic divide they fall. HDP has not been shown to be correct, but it has not been shown to be incorrect, either. Until it has, given that our intuitions on the matter are unreliable, the heavy duty platonist has no good reason to accept that some physical magnitudes are intrinsic, so can reject I1. The heavy duty platonist has good reasons for rejecting either I1 or I2, so the intrinsic argument fails.

3. Arguments from Crane

I now formulate two arguments against HDP alluded to by Crane. The first is that HDP must be coupled with one of the following unpalatable options: that there is a metaphysically privileged measurement scale; or that a physical magnitude is a case of an object being related to all numbers the magnitude property is measurable by. I will show that both options are defensible. The second argument is that HDP implies the following alleged contradiction: physical objects have some of their causal powers by being related to non-causal objects. I show that this is not contradictory, and outline a plausible theory of explanation that reveals why.

Crane argues that physical magnitude ascriptions involve an arbitrary choice of units that determines which number the physical object is said to be related to. In the case of temperature, the choice between using the relation *degrees Celsius* or the relation *kelvin* changes which number boiling water is said to be related to: it bears *degrees Celsius* to 100 or *kelvin* to 373.15. The number mentioned is determined by an arbitrary decision. This is taken to suggest that these predicates are just convenient ways of picking out the physical properties of physical objects (Crane 1990: 227). We get the following argument:

The arbitrary argument

A1: No measurement scale is such that the relation it specifies is fundamental.

A2: A physical object does not have a magnitude property in virtue of being related to all of the numbers the property is measurable with.

A3: Physical objects do not have physical magnitudes in virtue of a relation to numbers. [From A1 and A2]

A4: If HDP is true, objects have physical magnitudes in virtue of being related to a number or a collection of numbers.

A5: HDP is false. [From A3 and A4]

(See Daly and Langford 2009: 643 for a similar argument.) There are two ways to respond. The first is to undermine A1 by claiming that there is a metaphysically privileged measurement scale. There are two paths to take here: the bold, and the cautious. The former is to give reasons why an existing scale of measurement is metaphysically privileged; the latter is to claim that there is a privileged unit of measurement, though we may never be in a position to know which one it is. I will show that both paths are defensible.

For the bold path, there will have to be some virtue of using a certain scale that implies it has metaphysical import. Surprisingly, a candidate suggests itself: Planck units. As John Baez points out, our current physical worldview is 'deeply schizophrenic' (2001: 177). On the one hand we have the theory of general relativity, which recognises that spacetime is curved while ignoring the uncertainty principle. Two constants appear throughout: the speed of light c and Newton's gravitational constant G, which determines how much the geometry of spacetime is affected by other fields. On the other hand, we have quantum field theory, which takes the uncertainty principle seriously, but assumes that spacetime is flat. The two constants here are c and Planck's constant h, which defines the limitations governing our ability to measure simultaneously and two different quantities. On the face of it, these theories are incompatible, and some kind of reconciliation is needed if we are to fulfil science's goal of explaining all the phenomena in its domain.

Planck discovered a way to use the previously mentioned constants to define unique units of length, mass and time. Planck length is defined as $l_p = \sqrt{\frac{\hbar G}{c^3}}$ (where $\hbar = \frac{h}{2\pi}$), and is very small at about $1.61619910^{-35}\mathrm{m}$. There are several reasons for thinking that Planck length will play a significant role in our unified theory of everything. I will outline just one (but see Wilczec 2001a; 2001b; 2002 for more.)

According to quantum field theory, associated with every particle of mass m is its 'Compton wavelength' such that determining the position of the particle to within this length requires enough energy to create another particle of mass m. Thus, the Compton wavelength is the length at which quantum field theory becomes crucial for describing behaviour of particles of a certain mass (cf. Baez 2001: 179). According to general relativity, associated with any mass m there is its 'Schwarzschild radius' such that compressing an object of mass m to a size smaller than this results in the formation of a black hole. Thus, the Schwarzschild radius is the length at which general relativity becomes crucial for describing the behaviour of particles of a certain mass (cf. Baez 2001: 180). The Compton wavelength and Schwarzschild radius are equal when m is the Planck mass; the point at which they both equal the Planck length. As Baez says: 'At least naïvely, we thus expect that both general relativity and quantum field theory would be needed to understand the behaviour of an object whose mass is about the Planck mass and whose radius is about the Planck length' (2001: 180). This suggests that a unified physical theory will take Planck-sized chunks as its fundamental quanta. (Garay 1995 has also argued that a minimum length is a model-independent feature of all approaches to formulating a theory of quantum gravity.)

To claim that this scientific significance implies that Planck units are metaphysically privileged, only one more highly plausible claim is required: that the unified physical theory will take Planck-sized quanta as fundamental because the world is carved up into Planck-sized chunks. If it turns out that the unified physical theory represents Planck-sized quanta as fundamental, which looks likely, then there would be unique numerical values assigned to physical magnitude properties. The heavy duty platonist can happily reject A2 and the arbitrary argument.

Some may worry that the plausibility of HDP should not depend on the deliverances of as yet undeveloped physical theories. In which case, the cautious path should be taken. First, maintain that the physical world is made up of discrete quanta, and that physical properties can only take on certain discrete magnitudes that are functions of some smallest magnitude. Assuming that these smallest magnitudes are relations to the number one, each physical magnitude then has a unique numerical value, implying that some privileged scale of measurement accurately reflects the way the physical world is carved up. Second, accept that we may never be in a position to know which scale is privileged in this way.

One might object to the claim that the way world is carved up is potentially unknowable, but it is hard to see why this potentially unknowable truth should be objectionable. Unless, of course, one rejects the general claim that there is a determinate way the world is independently of whether we can know it, and so subscribes to some form of verificationism. I don't find this general claim implausible, and suspect that it is ubiquitous. Verificationism is no longer a widely held view, and for good reason. (See Lycan 2000: 115-128 for a good survey of the problems facing verificationism.)

The second way of responding involves altering HDP so that accepting that physical magnitudes are had in virtue of being related to every number the magnitude is measureable by is not absurd. For this to be successful, it must be shown that this is a defensible position. I will now demonstrate that it is.

What exactly is absurd about the claim that physical magnitudes are relations to all numbers? One worry is that individuating different magnitudes would be impossible. If mass related physical objects to just one number, different magnitudes of mass would be individuated by the number the objects are related to. According to the present view, however, every mass is a case of being related to every number. The other worry is that positing an infinite number of fundamental relations to explain a single property is metaphysically baroque. I will address each of these in turn.

Which number an object is related to does not tell us what physical magnitude property it has. However, which relations relate the object to which numbers does. Once a scale is set, it is not arbitrary which number an object is related to by the specified relation. This is how we individuate physical magnitudes on the present view.

To illustrate, we can represent each different mass with a function that takes all possible mass scales as its domain, and the real numbers as its range. The function representing one mass cannot be individuated from one representing another by its domain and range. However, which values of the range are assigned to which values in the domain will be different for each. For example, for a 5kg and a 10kg mass, the function representing the former maps *mass in kilograms* to 5, while the function representing the latter maps *mass in kilograms* to 10. These functions are distinct, and the mass properties they represent are distinct.

For the second worry, a stronger claim is required: physical objects are related to all numbers, but only indirectly; they are primarily related to the functions just described. Call this relation between physical objects and functions 'embodiment'. Mass scales can be described as relations assigning numerical values to each object with mass. On this account, what it is for *o* to have mass 10kg is for it to embody a certain function f that maps *mass in kilograms* to 10. If this is a tenable view, then the arbitrary argument fails. Though the physical object is related to all numbers, this is only in virtue of a more fundamental relation holding between the physical object and a function. Thus there is no absurdity, since the explanation for each property bottoms out on one fundamental relation.

Some objections spring to mind. The first is a charge of obscurity: we have not been told what embodiment amounts to. To this, the heavy duty platonist has a few things to say. The relations posited by HDP are supposed to be fundamental and not explainable in any other terms. The complaint that no further explanation has been given is therefore misguided. Nevertheless, a somewhat metaphorical gloss can be given of what the embodiment relation involves: an object's embodying f involves the object pairing up numbers and measurement scales in accordance with f.

Another objection is that, in its current form, HDP is too far removed from the naïve view described in §1. In responding to the arbitrary argument, the heavy duty platonist has given up on HDP. This worry is neutralised by recognising the following. First, the present view still characterizes physical magnitudes as fundamental relations between physical objects and mathematical objects. Second, it still characterises physical magnitudes as involving relations to numbers; it merely posits an intermediary relation. The present view is very much in the spirit of HDP.

The first argument from Crane was that HDP must adopt one of two incoherent claims: that there is a metaphysically privileged measurement scale; or that physical objects are related to all numbers by physical magnitude relations. I have shown that both of these claims are defensible. The first by either taking the scientific importance of Planck units as evidence of their being metaphysically privileged, or claiming that, though we may never be able to find out which one it is, there is nevertheless a privileged scale of measurement. I showed that the second claim can be defended by altering HDP slightly so that physical magnitudes are relations to certain unique functions from measurement scales to numbers.

I turn to the second argument from Crane. It is a commonplace view in the philosophy of mind that intentional states are relations holding between thinkers and propositions. It is also commonplace to suppose that intentional states are causally relevant to behaviour. But the combination of these views implies that a relation to an abstract object can endow a thinker with causal powers. Some philosophers find this problematic and have tried to undermine the view that intentional states are relational by appeal to analogy. First, it is assumed that the role numbers play in physical magnitude ascriptions is merely to index purely physical properties. Second, it is argued that this role is analogous to that played by propositions in intentional state ascriptions. The conclusion is that intentional states are not relational after all. (See Churchland 1979: 105 and Stalnaker 1987: 8 for similar lines of reasoning.)

If HDP is assumed instead, the same analogical reasoning supports the view that intentional states are relations to propositions. But Crane rejects HDP for the same reasons mentioned above: ‘How could the state of something’s having a certain temperature have effects, if it is really a relation to an abstract object?’ (1990: 225-226). This suggests a reductio argument against HDP:

The causal argument

Assume for reductio:

C1: Physical objects instantiate the physical magnitudes they do by bearing certain relations to numbers.

So, given that:

C2: Some physical objects have some of their causal powers by virtue of their physical magnitudes.

C3: Numbers are non-causal.

C4: No physical object can have causal powers by virtue of being related to something non-causal.

We get:

C5: Some physical objects have some of their causal powers by virtue of being related to something non-causal.

C5 contradicts C4. Hence, C1 is false.

To properly establish the reductio, it must be shown that the only way to avoid contradiction is to reject C1. C2 is well-motivated by some relatively non-contentious causal explanations in science that make indispensible reference to physical magnitudes. I will also assume the truth of C3: numbers are typically thought to be abstract objects, and abstract objects are characterised as non-spatial, and non-causal. Therefore, the only option for defending HDP lies in rejecting C4. As it happens, this is a poorly motivated premise. I will show that it is coherent and even quite plausible to assume that its relations to abstract objects can determine the causal powers of a physical object.

To illustrate, I will focus on the example of boiling water in a glass flask, assuming that it is paradigmatic of scientific causal explanations, and so can be easily generalised. According to Frank Jackson and Philip Pettit (1990: 109), there are two explanations available for why the glass breaks. The one given above mentions the temperature property. Jackson and Pettit call this the 'program explanation'. The other is called the 'process explanation': the glass breaks because a certain water molecule strikes some glass molecule with sufficient momentum to break its bonds. The fact that there are two explanations available poses a problem. If the behaviour of a certain water molecule is sufficient to break the glass, what use are the program explanation and the temperature property it mentions? It appears we either have to get comfortable with causal over-determination, or accept that the temperature property isnt causal. Neither seems desirable.

Any account of this explanation will have to reconcile these two competing explanations plausibly. This poses the following three challenges for HDP's account: (i) explain how the temperature property, understood as a relation to a number, is relevant to the breaking of the glass; (ii) explain how the momentum property of the water molecule, understood as a relation to a number, is relevant to the breaking of the glass; (iii) explain how both properties can be relevant, while avoiding causal over-determination.

Jackson and Pettit claim the threat of causal over-determination disappears once we appreciate the distinction between a causally relevant property and a causally efficacious one (1990: 114-7). Pettit explains: ‘a higher-order property is causally relevant to something when its instantiation ensures in a non-causal way, that there are lower-order properties present which produce it’ (1993: 37). The ‘ensuring’ relation is to be understood as a modal relation between properties: the causally relevant (program) property must always be accompanied by some lower-order property. The temperature property is a higher-order property that is a measure of the mean kinetic energy of the water molecules, and thus is determined by the distribution of momentum properties among the molecules. It is also multiply realisable: different distributions of momentum properties are also sufficient for its instantiation, so long as they produce the same average energy. Therefore, whenever this temperature property is instantiated by some body of water in a glass flask, there will always be one molecule or other that strikes the glass with sufficient momentum to break it. Though the temperature property is not causally efficacious, it is causally relevant to the breaking of the glass because it ensures the instantiation of an appropriate momentum property.

It is tempting to read ‘ensures’ as a metaphysically loaded term, such as ‘determines’. This must be resisted. Though it hasn’t been explicitly said in the literature, the ensuring relation cannot be interpreted as a determination relation. Because the temperature property is a higher-order property, multiply realisable by properties of molecules, it is clear that metaphysical determination runs in the opposite way to the ensuring relation here. (See Rosen 2010 and Audi 2012 for more on metaphysical dependence.) The temperature property is instantiated in virtue of the distribution of momentum properties among the water molecules. The ensuring relation must therefore be read epistemically: if we know that the temperature property is instantiated, we can be sure that one of the many ways it can be realised must obtain, and so we can be sure that the glass will break. It is the dependence of the temperature property on its realiser properties that explains why we can be sure of this.

It might initially seem that HDP cannot adopt this explanation of the relevance of the temperature property to the breaking of the glass. This is because, according to HDP, the relation between the water and the number is fundamental. To claim that it is also a multiply realisable higher-order property that ensures that some lower-order property is instantiated may seem inconsistent. But this appearance reveals itself as illusory, once the following is appreciated. The more fundamental momentum properties appealed to are properties of objects distinct from the body of water instantiating the temperature property: they are properties of water molecules, the constituents of the body of water. As stated at the end of §1, HDP implies that the relation between the water and a number is only fundamental relative to the water. This is perfectly consistent with the claim that the momentum properties of the molecules are ultimately more fundamental. Jackson and Pettit's story about the relationship between the two explanations is therefore available to the heavy duty platonist: the relation between a number and the water ensures that some water molecule or other instantiates an appropriate momentum property.

The heavy duty platonist can therefore meet (i) above, but cannot yet meet (ii) or (iii). Unlike Jackson and Pettit, the heavy duty platonist cannot claim that the momentum properties programmed for by the temperature property are causally efficacious because she considers these properties to be relations holding between water molecules and numbers. Thus HDP faces (ii): explain how the momentum property of the water molecule is relevant to the breaking of the glass. The model offered by Pettit and Jackson will be of no help here, for there are no properties of the constituents of the molecule to appeal to; and even if there were, appealing to them merely defers the problem. Sooner or later the heavy duty platonist will have to explain how a property of an object is relevant to the causal production of an event in terms of properties of that object. Thankfully, there is such an explanation.

The heavy duty platonist must identify some property of the water molecule, Q, such that Q is causally efficacious in the breaking of the glass. She will have to provide an account rendering it plausible that the momentum property of the water molecule, P, is metaphysically responsible for the instantiation of Q, and that Q is causally efficacious in the breaking of the glass. Jackson and Pettit's model cannot help because the ensuring relation runs from higher-order derivative properties to more fundamental properties. According to HDP, P is fundamental with respect to the water molecule, so Q must be understood as metaphysically derivative of it. My suggestion is that Q is the disposition *being prone to break glass*. It is plausible that dispositional properties are causally efficacious. (McKitrick 2005 has shown that, on the most plausible accounts of causality, dispositions are causal. See Mumford and Anjum 2011 for a theory of causation based on a metaphysic of dispositions.) And now we have a straightforward and plausible story to tell about why Q is dependent on P: objects are prone to break glass because they have one of many momentum properties, one of which is P. Moreover, the threat of causal over-determination disappears because there is only one property that causes the glass to break, namely the disposition; the momentum property non-causally determines that the molecule has the disposition, while the temperature property non-causally ensures the instantiation of the momentum property.

This account does not attribute any causal powers to the number the molecule is related to. The role it plays is to help determine, non-causally, that the physical object it is related to has a certain causally efficacious disposition. The thought that a relation a physical object stands in can endow that object with certain dispositions is not incoherent, even if the other relatum is an abstract object. Indeed, it is familiar: believing a proposition can dispose the believer to behave in certain ways. The apparent inconsistency of HDP was appealed to in order to undermine this commonplace view of intentional states. Having shown that HDP is not inconsistent, the heavy duty platonist is free to appeal to philosophy of mind to demonstrate that non-causal determination of effects by abstracta is widespread. For example, believing that red kidney beans contain a high concentration of toxin disposes one to boil them for ten minutes before using them in cooking. Frege puts the point as follows:

> How does a thought act? By being apprehended and taken to be true. This is a process in the inner world of a thinker which can have further consequences in this inner world of a thinker which can have further consequences in this inner world and which, encroaching on the sphere of the will, can also make itself noticeable in the outer world. If, for example, I grasp the thought which we express by the theorem of Pythagoras, the consequence may be that I recognise it to be true and, further, that I apply it, making a decision which brings about the accel-

> eration of masses. Thus our actions are usually prepared by thinking and judgement. And so thought can have an indirect influence on the motion of masses. (Frege 1956: 310)

It is quite plausible to hold that some physical objects have some of their causal powers by instantiating relations to non-causal entities. The heavy duty platonist has no reason to reject C1 and every reason to reject C4, rendering C5 harmless. The causal argument fails.[2]

4. Arguments from Field

Field (1989: 171-226) relies on the falsity of HDP to vindicate a substantivalist view of space-time, and undermine a relationalist view. He claims that the latter implies HDP and so is untenable. Why does Field think that HDP is so poisonous?

One argument Field offers is aimed at showing that HDP is inconsistent with both the letter and spirit of relationalism. The letter demands that only relations between aggregates of matter be posited in our fundamental theory. Yet HDP involves relations between aggregates of matter and numbers. The spirit of relationalism is expressed as the thought that 'only quite unproblematic relations will suffice' (1989: 192). Yet Field claims relations between physical objects and numbers are 'extremely problematic if not somehow demystified', and adopting HDP

[2]Balaguer appeals to the non-causal nature of mathematical objects to argue that what 'science says about the physical world could be true even if there aren't any mathematical objects' (1998: 133). This suggests a further argument against HDP. Numbers are non-causal, so the number 10 does nothing to make it that a 10kg brick is related to the number 10; therefore, there is something about the brick alone that does. I discuss this in a note because a reply can already be found in the literature: Baker rightly accuses Balaguer of sliding 'from the claim that the physical world is not causally dependent on the existence of the mathematical objects to the stronger claim that it is not dependent 'in any way' on their existence' (2003: 250).

involves claiming that such relations have no explanation.

This does not yet amount to an argument against adopting HDP, but only questions the coherence of a relationalist doing so. However, the spirit of relationalism suggests a plausible assumption about what our physical theories should be like from which a more general argument against HDP can be constructed.

The assumption we can take from the spirit of relationalism is this. Our physical theories should take only unproblematic relations as fundamental. Along with the premise that HDP employs problematic relations, this would count against HDP. Call this the 'problematic argument'.

The premises are not yet well motivated. Why is a physical object's being related to a number via a magnitude relation something that should puzzle us and require further explanation? Say an object has a mass of 10kg. We have knowledge of each relatum, albeit of a different sort, and we know what it is that relates them, namely *mass in kilograms*. We also know what the conditions are in which we can reliably say when the object stands in such a relation, and know what behaviour this determines in the object. What exactly is it that needs explaining here? We need to know what the unproblematic relations are, and why they do not require further explanation. Take the causal relation. This is often considered innocuous by nominalists such as Field, but I fail to see that our understanding of the causal relation goes any further than what I have just said about mass in kilograms. The problematic argument is not successful.

Field motivates one further objection to HDP by drawing attention to the fact that it must explain the behaviour of physical systems in terms of relations between physical objects and numbers. Such explanations, he claims, are extrinsic 'because the role of numbers is simply to serve as labels for some of the features of the physical system' (1989: 193), and undesirable because, whenever one has such an explanation, 'one wants an intrinsic explanation that underlies it: one wants to be able to explain the behaviour of the physical system in terms of the intrinsic features of that system' (1989:193). Call this the 'explanatory argument'.

Elsewhere, Field specifies that, by 'intrinsic features' of a physical system, he means those features that are causally relevant to its behaviour (1989: 18). The response to the causal argument in §3 is enough to see off this objection: though HDP has physical objects related to non-causal objects, those relations are nevertheless relevant to the causal behaviour of physical objects.

One could suggest a different interpretation of 'intrinsic explanation': an explanation that only mentions intrinsic properties of the system, or the objects that comprise it. In which case, the response to the arguments from Lewis outlined in §2 will be enough to avoid this objection. According to a plausible and popular analysis of intrinsicality, the properties of physical objects posited by HDP are intrinsic. If the proponent of the explanatory argument is loath to accept the Lewisian analysis, the heavy duty platonist can point out that our intuitions regarding which properties are intrinsic are misleading anyway. The proponent of the explanatory argument will surely accept explanations of physical behaviour in terms of mass; but we have seen that and object's mass involves something distinct from that object.

Whichever interpretation is endorsed, the failure of the explanatory argument is ultimately due to the same error. The view that the role of mathematics in explanations is simply to label physical features is assumed on behalf of the heavy duty platonist; but the heavy duty platonist would not, and must not, assume this view. Rather, she has it that there is a robust metaphysical connection between physical objects and numbers that renders the latter explanatorily relevant to physical phenomena.

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