What 'The number of planets is eight' means[*]

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Abstract

'The following sentence is true only if numbers exist: (1) The number of
planets is eight. (1) is true; hence, numbers exist.' So runs a familiar argu-
ment for realism about mathematical objects. But this argument relies on a
controversial semantic thesis: that in (1) 'The number of planets' and 'eight'
are singular terms standing for the number eight, and the copula expresses
identity. This is the 'Fregean analysis'. I show that the Fregean analysis is
false by providing an analysis of sentences such as (1) that best explains the
available linguistic data, and according to which no terms in sentences such
as (1) purport to stand for numbers.

1.   Introduction

'The following sentence is true only if numbers exist:

    (1) The number of planets in the solar system is eight.

(1) is true; hence, numbers exist.' This is a familiar 'easy argument' for realism
about mathematical objects. (I will henceforth omit 'in the solar system' from (1);
consider the restriction to the solar system implicit.) The easy argument assumes a
substantial semantic thesis concerning sentences such as (1): that expressions such

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as 'The number of planets' (*number-of expressions*) and 'eight' (*cardinals*) are referring expressions that stand for numbers, and that the copula expresses identity. This analysis is derived from Frege ([1884] 1953: 69), so call it the 'Fregean analysis'.

The Fregean analysis has great philosophical import not only for ontology, but also the philosophy of language. As Thomas Hofweber (2005: 179-80) notes, the analysis gives rise to a linguistic puzzle. In (1), 'eight' appears to occur in singular-term position, reserved for referring expressions. However, associated with sentences such as (1) are sentences which appear to have the same meaning, and yet contain cardinals in adjectival position, reserved for non-referring terms:

(2) There are eight planets.

There is a syntactic and semantic dimension to this puzzle. The former arises because singular terms cannot typically occur in adjectival position grammatically. The latter arises because, intuitively, the adjectival occurrences of cardinals do not serve the same semantic function as their singular-term counterparts. The latter appear to stand for objects while the former do not. Yet the similarity in meaning between (1) and (2) means it is implausible to suggest that 'eight' in (1) and 'eight' in (2) are merely homonyms. I follow Hofweber (2005: 180) in calling this syntactico-semantic puzzle 'Frege's Other Puzzle' (FOP).

The Fregean analysis gives rise to a linguistic puzzle and appears to yield controversial ontological conclusions. Whether or not this analysis is correct should therefore be of great concern to philosophers. Recently, several authors (Hofweber 2005; Moltmann forthcoming; Felka 2014) have offered rival analyses of sentences such as (1), each claiming that their analysis is to be preferred to the Fregean ana-

lysis because it solves FOP. However, a rival only truly solves FOP and bears on the ontology of mathematics if it provides the best explanation of all the other available linguistic data. The primary aim of this paper is to provide an analysis that does just that.

In §2, I argue that cardinals in adjectival position are adjectives. In §3, I endorse the claim that sentences such as (1) are specificational sentences. However, I show that the popular analysis of specificational sentences as question-answer pairs suffers serious drawbacks: it cannot plausibly explain the differences in distribution of free relative clauses and definite descriptions, both of which occur as the subjects of specificational sentences. In §4, I motivate analyses of free relatives and definite descriptions, as they occur in specificational sentences, that explains their difference in distribution. In §5, I present my own account of specificational sentences that solves FOP and suffers none of the drawbacks of its rivals. I conclude in §6 that the Fregean analysis is mistaken, along with the other rivals to my analysis, and that the easy argument for mathematical realism is unsound. I discuss options for salvaging the argument, but the lesson that emerges is that establishing the existence of mathematical objects is not easy.

2.   Adjectiveal Cardinals

Call cardinals in adjectival position 'cardinal modifiers'. Hofweber (2005) argues that cardinal modifiers are determiners. In this section, I outline the case for this position. Though it is initially promising, I argue that the truth is more complicated. Cardinal modifiers are adjectives. They can sometimes occur in unexpected contexts; but I show that independently motivated mechanisms in natural language

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can explain their unusual distribution. The result is a plausible and unified semantic analysis of cardinal modifiers that sets the agenda for solving FOP.

Analysing natural language using the first-order quantifiers is problematic. Natural language contains compound quantifier phrases: 'Some apples'; 'Every man'; 'Most coins' etc. This suggests 'Something' and 'Everything' are also compound. An adequate analysis of natural language should provide a unified and systematic analysis of quantifier phrases (cf. Hofwever 2005: 196), and account for all quantifier phrases of the language (cf. Barwise and Cooper 1981: 156-61).

Generalized Quantifier Theory (GQT) is a plausible and widely-accepted means of providing such an account. (GQT is based on the work of Mostowski 1957 and Montague 1974; for an overview of its development, see Keenan and Westerstahl 1997.) According to GQT, sentences are typically composed of a noun phrase (NP) and a verb phrase (VP). Each are assigned semantic values in accordance with their semantic type. VPs are typically assigned type $< e, t >$, or functions from individuals (type e) to truth-values (type t), and are assigned sets of individuals as their semantic values. NPs are either quantifier expressions of type $<< e, t >, t >$ (functions from sets to truth-values), or names of type e (individuals), and are assigned sets of sets of entities or individuals, respectively.

Sentences are of type t and their semantic values are truth-values. The truth-values of sentences are determined compositionally by functional application. When interpreting a sentence S containing a quantifier expression as NP, for example, the semantic value of the NP takes the semantic value of the VP as an argument and yields a truth-value. To illustrate, for 'All red peppers grow quickly' the set of things that grow quickly is the semantic value of the VP, and the function that yields 'true' for all and only sets that contain all of the red peppers is assigned to

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the quantifier expression. Thus, the sentence is true in iff the set of things that grow quickly contains all the red peppers—iff all red peppers grow quickly. The above outline of GQT concerns only the extensional semantic values assigned to expressions. Expressions are also assigned intensions, which are functions that take a possible world as an argument to yield the expression's extension in that world.

Before continuing, a quick note on the difference between denotation and reference. In formal semantics, set-theoretical objects are assigned to expressions as semantic values. It must be stressed that these semantic values are in an important sense arbitrary. Beyond their playing the right role in the formal system for generating precise statements of truth-conditions, they have little significance. I call the relation an expression bears to its semantic value 'denotation' and the relation an expression bears to the object in the world it is systematically used to pick out 'reference'. To illustrate, for various reasons, Montague treated proper names as quantifier expressions. On this view, 'David Attenborough' denotes the set of all the sets that have David Attenborough as a member; but the name does not refer to this set, it refers to the person. The referent of an expression must bear a suitable relation to the semantic value of the expression so that the truth-conditions come out correctly overall, but that relation need not be identity.

In quantifier expressions, the modifying elements are determiners. 'Some' in 'Some apples' modifies 'apples' to yield an expression that concerns one or more apples. Other determiners include 'Most', 'Every' and 'Both'. The semantic type of a noun is $< e, t >$ so its extension is a set. The semantic type of a determiner is $<< e, t >, << e, t >, t >>$ and its extension is a function from sets to functions from sets to truth-values—a function from sets to the extensions of quantifier phrases.

According to GQT, number terms in adjectival position are determiners. For example, 'Six' modifies 'apples' to produce the quantifier expression 'Six apples'. GQT is a plausible, unified and widely-accepted treatment of determiners and quantifier expressions in natural language, and its account of cardinal modifiers makes some accurate predictions. Consider the following:

(3) Four pigs surrounded the house.

The NP here is clearly behaving as a quantifier expression, which suggests that the cardinal modifier is behaving as a determiner. However, such examples do not exhaust the syntactic distribution of cardinal modifiers. They can also occur as adjectives. The semantic type of an expression is supposed to determine the semantic values assigned to it and which expressions it can be meaningfully combined with. Determiners are of type $<< e, t >< < e, t >, t >>$, so they can only take expressions of type $< e, t >$ as arguments. A determiner should not therefore be able to take a quantifier phrase as an argument. There are examples that verify this prediction:

(4a) ? The all men in the room wore silk.

(4b) ? All some chickens were cooked.

However, there is a particular subclass of 'determiners' that combine with nouns to form expressions that can serve as arguments for determiners. Cardinal modifiers in particular exhibit this pattern. Consider:

(5a) The few/many/four berries in the basket were squashed.

(5b) All four berries in the basket were squashed.

(5c) Some four students turned up today.

Here, 'few/many/four berries' cannot be quantifier phrases because they occur as arguments for 'The', 'Some' and 'All', which are determiners. The modifiers 'four', 'many' and 'few' cannot therefore be determiners in this context. They modify a noun to make something that might serve as an argument for a determiner: another noun. This suggests that the modifiers occur here as adjectives of type $<< e, t >, < e, t >>$, and are assigned functions from sets to sets.

There are two further pieces of evidence suggesting that cardinal modifiers can occur as adjectives. The first is that complex cardinals appear to get their meaning compositionally. Consider:

(6a) Mary saw two hundred pigs.

(6b) Mary saw two thousand pigs.

Intuitively, 'two' makes the same contribution to the meanings of (6a) and (6b), the difference in meaning being due to the difference in meaning between 'hundred' and 'thousand' (cf. Ionin and Matushansky 2006: 317). These intuitions are honoured if we assume that cardinal modifiers are adjectives. Following Godehard Link (1983), assume that the plural noun 'pigs' denotes a set whose members are all the pluralilties of pigs. A pluralily of pigs is a plural object that is divisible into non-overlapping parts, all of which are pigs. In (6a), 'hundred' modifies 'pigs' and yields the noun 'hundred pigs' which denotes a set containing all and only the pluralities that are divisible into pluralities of pigs that are divisible exactly into one hundred pigs (including the plurality that is one hundred pigs). Then, 'two' modifies this to yield a noun denoting a set containing only those pluralities divisible into two non-overlapping pluralities which are divisible exactly into one

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hundred pigs. The story for (6b) would be exactly the same but the modifier 'thousand' would yield a noun denoting a set containing pluralities divisible exactly into two pluralities which are divisible exactly into one thousand pigs. In both, 'two' makes exactly the same contribution, and the difference between (6a-b) is rightly attributed to the difference in meaning of 'thousand' and 'hundred'.

In contrast, the traditional GQT story treats 'Two hundred' and 'Two thousand' as distinct determiners, the meanings of which are not compositionally determined. This neither captures the similarity between the contributions made by 'two' in (6a-b), nor the difference between the contributions made by 'hundred' and 'thousand'. A compositional account of compound cardinals requires they be analysed as adjectives (see Ionin and Matushansky 2006 for a developed analysis).

Another datum comes from apparent adjectival uses of combinations of cardinals and nouns. The verb 'considered' should only be followed by the attribution of a property to something (cf. Partee 1986: 361): one considers something to be a certain way. Moreover, the conjunction 'and' can only combines expressions of the same semantic type. Now consider:

(7) Mary considered her lunch two portions and cheap!

Here 'two portions' is both the complement of 'considers' and part of a conjunction in which one of the conjuncts is the noun 'cheap'. This suggests that it too is serving as noun, and so 'two' is serving as an adjective. There are two strong pieces of evidence showing that cardinal modifiers can occur as adjectives.

There is evidence that cardinal modifiers can occur both as determiners and as adjectives, but it is implausible to suggest that they are systematically ambiguous. There must be a common thread of meaning that unites their use in sentences such

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as (3), on the one hand, and (5), (6) and (7), on the other. But how else can we explain the evidence on the present framework, which assigns a single semantic type to each expression type? Before answering, I will demonstrate that the puzzling syntactic distribution of cardinal modifiers is not unique. Many adjectives exhibit an identical pattern. Consider:

(8a) She considered them tiny insects and invertebrates.

(8b) Tiny insects surrounded the house.

The fact that 'tiny insects' can form a conjunction with the noun 'invertebrates' and occur as the complement of 'considered' in (8a) is good reason to take it to be a noun, too. In (8b), however, 'Tiny insects' is the subject of the sentence, forming a full NP with existential force. How are we to explain this?

There is a modification GQT that is both intuitive and explanatory of the present data. The idea is to reject the one-to-one correspondence between expression types and semantic types, and instead assign to each expression, as well as its base semantic type, a family of other types that it can take on if the linguistic context demands it. This does not mean giving up on compositionality. Theorists have uncovered systematic rules that govern this kind of type-shifting (see Partee and Rooth 1983 and Partee 1986). For instance, it is generally agreed that type-shifting occurs only as a last resort to avoid a type miss-match. A plausible rule specific to plural nouns is that, when combined with a VP that implies spatial location, their type is raised to that of a quantifier expression. This explains why 'Tiny insects' (8b) has existential force.

Given the similarity in distribution between adjectives and cardinal modifiers, I propose that they be understood in the same way: they standardly occur as ad-

jectives, but, subject to specific linguistic contexts in which a type miss-match would otherwise occur, the nouns they form part of are type-shifted to form quantifier expressions. This explains the otherwise puzzling syntactic distribution of cardinal modifiers. On this view, cardinal modifiers do not themselves undergo type-shifting; only the nouns they form part of do. In 'Four pigs surrounded my house', the threat of type miss-match only looms when the noun 'Four pigs' and the VP 'surrounded my house' are merged. At this point, the whole noun 'Four pigs' is type-shifted to form a quantifier expression. The cardinal modifier never behaves as a determiner. Similarly, though 'Tiny insects' in (8b) carries existential force, it would be wildly implausible to suggest that 'Tiny' acts as a determiner in this sentence. This highlights a significant benefit of the present view: it provides a uniform semantics of cardinal modifiers. They only ever behave as adjectives, and their unusual syntactic distribution is explained by independently motivated rules governing plural nouns in general.

Now we have the semantics of cardinal modifiers, the agenda for solving FOP is set: the semantic and syntactic relation between cardinals in adjectival position and cardinals in singular-term position must be accounted for. If it can be shown that, despite appearances, cardinals in contexts such as (1) are adjectives, then FOP will have been solved. Pursuing this strategy involves understanding of the syntactic and semantic profile of sentences such as (1). In the following section, I argue that such sentences are specificational sentences, but show that the mainstream analysis of specificational sentences is subject to serious drawbacks.

3. Specificational Sentences

Some sentences are useful for emphasising certain aspects of the information they convey. Consider the following:

(9a) John swims quickly to shore.

(9b) The way John swims to shore is quickly.

Intuitively, the information conveyed by (9a-b) is the same. However, (9a) presents the information neutrally, while (9b) emphasises the way John swims. To illustrate, (9a-b) are both appropriate answers to the question 'How does John swim to shore?', but to the question 'Where does John swim to?', only (9a) is. This is an example of *focus*. As Hofweber (2005: 210-1) notes, sentences such as (1) exhibit focus. Consider again (1) and (2):

(1) The number of planets is eight.

(2) There are eight planets.

Though the information conveyed by (1) and (2) is intuitively very similar, (1) emphasises how many planets there are. To the question 'How many planets are there in the solar system?' both are appropriate answers; to the question 'What revolves around the sun?' only (2) is.

Sentences that exhibit focus and appear to specify which thing is mentioned by the subject of the sentence are called specificational sentences. To appreciate how specificational sentences differ from subject-predicate sentences, contrast the following from Line Mikkelsen (2011: 1806):

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Predicational:

(10a) The hat I bought for Harvey is big.

(10b) What I bought for Harvey is big.

Specificational:

(11a) The director of Anatomy of a Murder is Otto Preminger.

(11b) Who I met was Otto Preminger.

In (10a-b), the predicate 'is big' describes the thing picked out by the subject of the sentences, rather than specifying which thing it is. In (11a-b), however, the post-copular expression specifies which thing the pre-copular clause stands for.

Examples of predication and specification share syntactic features. (10b) and (11b) begin with an interrogative pronoun as part of a free relative. Both are 'pseudoclefts'. There are predicational and specificational pseudoclefts. (10a) and (11a) begin with a headed relative clause and are plain predicational and plain specificational sentences, respectively. Linguists agree that the distinction between predication and specification is the more semantically important one. In the absence of any evidence that different kinds of specificational/predicational sentences determine their truth-conditions differently, it is desirable to have a semantics of predicational sentences that unifies all their syntactic forms, including plain and pseudocleft, and a semantics of specificational sentences that unifies to all theirs (cf. Mikkelsen 2011: 1807; Felka 2014: 8).

The fact that sentences such as (1) exhibit the characteristic features of specificational sentences suggests that such sentences are specificational (cf. Felka 2014).

This is a promising avenue because many specificational sentences routinely have adjectival modifiers in post-copular position:
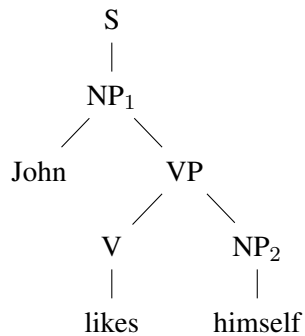
(12) The colour of the squirrels is red.

To see what contribution the post-copular expressions in sentences such as (1) make, specificational sentences must be analysed. To begin with, there is good evidence for taking the post-copular expression of a specificational sentence to be elliptical for a clause: doing so solves an important syntactic puzzle. Consider:
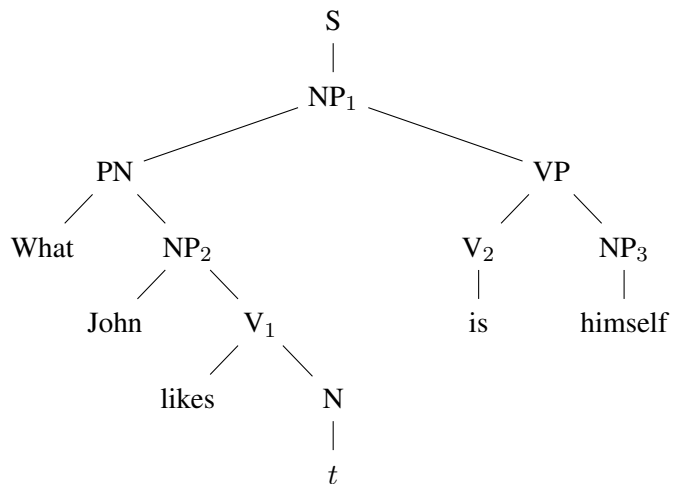
(13a) John likes himself.

(13b) What John likes is himself.

In both, 'himself' borrows its referent from 'John'. However, this is only possible if the two expressions stand in a specific syntactic relationship—*c-commanding*. This relationship is represented in syntax trees in the following way. A node x dominates a node y iff x is above y and one can trace a line from x to y while only moving downwards. A phrase a c-commands for a phrase b iff the first branching node dominating a also dominates b, and neither a nor b dominate each other. To illustrate, consider the syntax tree assigned to (13a):

```
                          S
                          |
                         NP₁
                        /    \
                    John      VP
                             /    \
                            V      NP₂
                            |       |
                          likes   himself
```

According to this tree, 'John' c-commands 'himself', so the former can lend its reference to the latter. Now contrast the syntax tree assigned to (13b), where 'John' does not c-command 'himself', so, it appears, the former cannot lend its reference to the latter. Yet it clearly does.

```
                              S
                              |
                             NP₁
                           /      \
                        PN          VP
                      /    \       /    \
                  What      NP₂   V₂     NP₃
                          /    \   |       |
                      John     V₁  is    himself
                              /   \
                          likes    N
                                   |
                                   t
```

If we accept that the post-copular expression is elliptical for a clause, the puzzle can be solved. Compare:

   (13b) What John likes is himself.

(13c) What John likes is John likes himself.

Here the post-copular expression 'himself' is elliptical for 'John likes himself', which shares the syntactic structure of (13a) and therefore has 'John' c-commanding 'himself'. The puzzle is thereby solved.

There is more evidence in support of the claim that the post-copular expression in specificational sentences is elliptical, the most impressive of which is that certain specificational sentences allow for the post-copular clause to be stated in full:

(14a) What I did then was call the grocer.

(14b) What I did then was I called the grocer. (Ross 1972, (39a), (19b))

Here the ellipsis is optional. Sometimes it is not, but there is a plausible explanation for this. Consider:

(15a) Who I called was the grocer.

(15b) *Who I called was I called the grocer.

The repetition of the verb makes ellipsis obligatory, while there is no danger of repetition of the verb in (14b) (cf. Schlenker 2003: 14).

Specificational sentences exhibit two kinds of pre-copular clauses. The first kind is a relative clause headed by an interrogative pronoun (a free relative). The second is a definite description, sometimes in the form of a headed relative clause. As I mentioned above, in the absence of evidence suggesting they should be treated differently, it is desirable to have a unified semantics of these different syntactic forms. How can we analyse the pre-copular clauses of specificational sentences in a way that meets this constraint? The following evidence suggests a promising means.

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(16a) The murderer of Smith is John.

(16b) Who the murderer of Smith is is John.

Notice that, intuitively, (16b) provides an adequate paraphrase of (16a). Similar evidence has often been appealed to (in Grimshaw 1979, for example) in defence of the view that definite descriptions are disguised free relatives in embedded contexts:

(17a) I know the capital of Italy.

(17b) I know what the capital of Italy is.

Given these, it is tempting to analyse definite descriptions in specificational contexts as free relatives in disguise. Free relatives are standardly classified a kind of interrogative clause. To the direct question 'Who is the murderer of Smith?', there is the indirect question 'Who the murderer of Smith is'. This suggests an analysis of specificational sentences according to which the pre-copular clause is a question to which the post-copular clause provides the answer in the form of an elided declarative sentence. Call this the 'question-answer analysis' (QAA). This is a popular analysis, and many different versions of it have been proposed (e.g. Moltmann forthcoming, Felka 2014, and Schlenker 2003). With respect to analysing sentences such as (1), the most compelling analysis is Katharina Felka's:

(18) [What the number of planets is] is [There are eight planets.]

According to Felka, what look like definite descriptions in contexts such as (1) are elided free relatives that pose questions, and what look like singular terms in post-copular position are elided declarative sentences that provide the relevant answer.

This analysis promises solve FOP because it takes the post-copular expression to be elliptical for an expression in which the cardinal modifier occurs in its typical syntactic position and contributes its typical semantic value.

However, there are two significant drawbacks to QAA that demand we look for an alternative. They suggest that the semantics of free relatives and definite descriptions, as they occur in specificational sentences, are distinct, meaning that a unified account such as QAA is inappropriate.

The first drawback is that headed relatives and the definite descriptions they are supposed to elide are semantically distinct when they occur outside of specificational contexts. In intensional contexts, free relatives can occur with the same meaning in both factive and non-factive contexts, while the corresponding definite descriptions can do so in factive contexts:

Non-factive:

(19a) I wonder/fear who the woman who John is seeing is.

(19b) *I wonder/fear the woman who john is seeing.

Factive:

(20a) I know/remember what the capital city of Italy is.

(20b) I know/remember the capital city of Italy.

This suggests a difference in the semantics of free relatives and definite descriptions in intensional contexts. It is of course open to the proponent of QAA to claim that only definite descriptions occurring in specificational sentences are elided free relatives, but with this QAA begins to feel ad hoc. The second drawback exacerbates this feeling. The problem is that the distribution of free relatives outstrips

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that of definite descriptions in specificational contexts. One example is that free relatives can form specificational sentences that specify the qualities an individual has, while definite descriptions cannot. Consider:

(21a) What the woman who John is seeing is is small.

(21b) The woman who John is seeing is small.

The truth-conditions of these sentences are intuitively the same, yet (21a) is specificational and (21b) is predicational, so the latter cannot be elliptical for the former. Another difference in distribution is demonstrated by the following:

(22a) Where the woman who John is seeing is is Scotland.

(22b) *The woman who John is seeing is Scotland.

Despite the contextual prompting of (22a), the definite description in (22b) cannot be elliptical for the free relative in (22a). Yet, if QAA is true, there is no good reason why definite descriptions could not occur as elided 'Where'-questions as well as 'What'- or 'Who'-questions (cf. Frana 2007). Again, the proponent of QAA could modify her position to the following: definite descriptions are elided free relatives only if they occur in subject position of specificational sentences that specify who or what an individual is. Now QAA looks extremely ad hoc.

In this section, I have argued that sentences such as (1) are specificational. I showed that, though QAA is an initially plausible means of providing a unified semantic account of the different syntactic forms of specificational sentences, the syntactic distribution of free relatives and definite descriptions in specificational and non-specificational contexts suggests that such a unified analysis is inappropriate. A better account of specificational sentences should attribute different se-

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mantics to free relatives and definite descriptions, but explain why their contribution to specificational contexts in which they both occur yields the same truth-conditions.

4.   Free Relatives and Definite Descriptions

In this section, I independently motivate accounts of both definite descriptions and free relatives in intensional and specificational contexts. I show that these accounts predict their differences in distribution, while still allowing a somewhat unified semantic account of specificational sentences.

I start by appealing to data concerning certain intensional transitive verbs. There is strong evidence to suggest that 'need' and 'want' take proposition-denoting expressions as their object (see Dikken, Larson and Ludlow 1997):

(23) The scientists need some new computers, but their budget won't
        allow it.

What the scientists need is that they get some new computers. The anaphoric 'it' is clearly referring to a proposition; reference to the scientists would instead require 'them'. This evidence is symptomatic of a general pattern. The impersonal pronoun 'it' is in general not appropriate for denoting people or properties.

(24a) *Jane called Jim and asked it its opinion.

(24b) *Jane said Jim is reproachable, and he is it.

However, 'it' is routinely used to refer to propositions and facts:

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(25a) It's true that there are Aliens, but the government is covering it up.

(25b) The fact that 2+2=4 is obvious; it can be proven.

It is therefore telling that specificational sentences concerning people only allow for the impersonal pronoun:

(26a) The winner of the competition is Anna, isn't it?

(26b) *The winner of the competition is Anna, isn't she?

(26c) Who the winner of the competition is is Anna, isn't it?

(26d) *Who the winner of the competition is is Anna, isn't she?

This suggests that the things referred to in specificational sentences are propositions or facts. But which is it? Consider:

(27a) What I love is a secret.

(27b) The thing I love is a secret.

Suppose I love mountain climbing. It doesn't make sense to say mountain climbing is a secret; it's the fact that I love mountain climbing that is secret. However, the same free relative and definite descriptions can also refer to a particular activity:

(28a) What I love is dangerous.

(28b) The thing I love is dangerous.

The fact that I love mountain climbing isn't dangerous; the activity is. How can we explain these two readings? I will consider free relatives first.

To account for both readings of free relatives, I will consider their syntax. The standard account is that they are syntactically derived from sentences by a process called 'movement'. In 'What I love' the pronoun 'What' has been moved out of its object position, as the complement of 'love', into subject position. This is represented as follows:

(29) What$_1$ I love _____$_1$.

On the copy theory of movement, revived by Noam Chomsky (1995), moved expressions leave behind a phonetically deleted copy of themselves in their original position. This view has a great many theoretical virtues. First, copying can be understood in terms of the procedure that combines two syntactic objects into a new syntactic object—*Merge*. A syntactic item is merged for a second time in a different place (movement is re-Merge). Second, it does not posit any other objects over and above lexical items, bringing us closer to a theory of syntax as a recursive procedure that operates only on the lexicon.

More importantly, the copy theory of movement leaves open the possibility of interpreting the moved item in its surface position and/or its original position. The following demand the moved item be interpreted in different positions (cf. Bhatt 2002: 56-57):

(30a) The first book$_i$ that John said Tolstoy ever wrote _____$_i$
       was Anna Karenina.

(30b) The first book$_i$ that John ever said Tolstoy wrote _____$_i$
       was Anna Karenina.

The reading we get from (30a) is that John said the first book Tolstoy wrote was

Anna Karenina, which is only possible if 'first book' is interpreted inside the scope of 'said'. The copy theory of movement makes sense of this: the original copy of 'first book' is interpreted while the surface copy is not. The reading of (30b), however, is that John often attributes works to Tolstoy, but on the first occasion he attributed Anna Karenina. For this, 'first book' must be interpreted outside of the scope of 'said'. The copy outside of the scope of 'said' is interpreted while the others aren't. (See Fox 2002 for a well-motivated account of how different copies are interpreted.)

The copy theory of movement explains the two readings of free relatives. In 'What I love', there is a choice as to which copy of the moved element 'What' we interpret. If the surface copy is interpreted, it is interpreted as an NP in subject position, and so refers to something. The embedded sentence 'I love ____' then acts as a modifier, narrowing the referent of the pronoun to the thing loved by me. If we interpret the original copy, then its semantic function is that of an NP in object position. In this case, the semantic contributions of the expressions in the free relative are the same as in the sentence it is derived from, 'I love what'. In this case, 'What I love' is a nominalization of 'I love what'.

Nominalizations are referring expressions derived from non-referring expressions that refer to something semantically assigned to the expression they are derived from. For example, the nominalization of the VP 'is bad' is 'being bad' which refers to the property expressed by the VP. Sentence nominalizations include 'that'-clauses. In 'John believed that there are aliens', the 'that'-clause nominalization 'that there are aliens' plausibly refers to the proposition that there are aliens. Propositions are semantically assigned to sentences as their intensions, so that nominalizations of sentences can refer to them is to be expected.

However, some sentence nominalizations appear to refer to facts or states of affairs. For example, the 'that'-clause in 'John remembered that it was Friday' plausibly refers to the fact that it was Friday, and the imperfect nominal of 'John's walking to the park' plausibly refers to the relevant state of affairs. In the present semantic framework, facts and states of affairs are not assigned to sentences, but it seems they should be if we want to give a systematic account of nominalizations.

Accounting for the referents of certain nominalizations is one of many reasons for introducing facts or states of affairs into the semantics of natural language (see Kratzer 2014 and references therein for more reasons). There are many proposals, perhaps the most famous of which is situation semantics (the classic text is Barwise and Perry 1983). There is not space here to motivate a particular account. It will suffice to make a small extension of the present framework (inspired partly by Moffett 2003, though he extends a different framework).

Instead of truth-values, I will assign facts as the extensions of sentences. The role of truth-values in formal semantics is to have one or the other assigned to a sentence in virtue of the semantic values of the parts of the sentence and the way they are composed. When things go well, a sentence is assigned True; otherwise it is assigned False (the metaphysical neutrality of this role is reflected in the fact that many use 1 and 0 as the semantic values of sentences). This role can be preserved by facts, so no harm is done to the truth-conditional framework. Instead of directly characterising the truth or falsity of sentences, I will indirectly characterise it in the following way. For each world w, there are two sets of facts, TRUE_w and FALSE_w. The former contains all the facts that are parts of w. They are the facts that, relative to w, obtain. The latter set contains all the facts that are not parts of w. These are the facts that do not obtain. So, a sentence is true at w iff it denotes a member of

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TRUE$_w$ and false iff it denotes a member of FALSE$_w$. On this view, the intensions of sentences, or propositions, are functions from worlds to facts, or, equivalently, sets of facts.

Aside from complicating the statement of truth-conditions, this yields a systematic account of the referents of sentence nominalizations. Depending on the context and the kind of nominalization, they can refer either to the extension of the relevant sentence (a fact), or the intension (a proposition). Now consider the following:

(31) What I want reflects badly on my character.

Suppose I want ice cream. It makes no sense to say that ice cream reflects badly on my character. This forces the original copy of the pronoun 'What', and not the surface copy, to be interpreted. Hence, the free relative is interpreted as a nominalization of the sentence 'I want what' and refers to the fact denoted by this sentence: the fact that I want ice cream.

We have seen that definite descriptions can also be used to refer to facts. The account I propose for them is inspired by a recent extension of Angelika Kratzer's (2002) *de re* account of knowledge ascriptions, proposed by Ilaria Frana (2006). Kratzer's analysis is that ascriptions of the form '*A* knows that p' ascribe knowledge *de re*, of some particular fact, that it exemplifies the relevant proposition. Intuitively, a fact exemplifies a proposition just in case it involves only things that are relevant to the truth of the proposition. On Krazter's view, 'that'-clauses play a dual role: they characterise a particular fact, and they characterise the content of the state of knowledge the agent has concerning that fact (Kratzer 2002: 659).

Frana's innovation is to recognise that this view can be extended so that things

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other than facts are the *res* of knowledge ascriptions. Consider:

(33) Ben knows the capital of Indonesia.

The definite description here plays an analogous dual role. First, it characterise a particular object, namely Jakarta; second, it gives the description correctly ascribed to Jakarta by Ben. (33) says that Ben knows, of Jakarta, that it is the capital of Indonesia. Both of these roles are unquestionably played by definite descriptions in other contexts: they correspond to the *de re* and *de dicto* readings definite descriptions unquestionably exhibit. The crucial claim here is that in some contexts they play both roles at the same time.

When a definite description plays a dual role, it can plausibly refer to a fact: the fact that the object characterized is the way the description says. Indeed, knowing of Jakarta that it is the capital of Indonesia is tantamount to knowing the fact that Jakarta is the capital of Indonesia. This makes perfect sense of the fact-referring occurrence mentioned above:

(34) The thing I love is a secret.

It makes little sense to say that mountain climbing is a secret, so the dual role of the definite description is triggered, and the fact that I love mountain climbing is referred to. This yields the desired interpretation.

The accounts of free relatives and definite descriptions I have developed can explain the difference in distribution between the two, demonstrated in the previous section. Recall that free relatives preserve their meaning across both factive and non-factive contexts, while definite descriptions and headed relatives can only occur in factive contexts:

Non-factive:

(19a) I wonder/fear who the woman who John is seeing is.

(19b) *I wonder/fear the woman who john is seeing.

Factive:

(20a) I know/remember what the capital city of Italy is.

(20b) I know/remember the capital city of Italy.

On the present account, it is expected that definite descriptions cannot occur in non-factive contexts. Assuming they take on their dual role in such contexts, (19b) would read that I wonder/fear of a particular woman whether she is the one John is seeing. If this makes sense, it is the wrong interpretation. More generally, the dual role of definite descriptions, and the resulting *de re* readings, imply some kind of acquaintance with the relevant object, whereas intensional verbs such as 'wonder' imply ignorance. As for how free relatives can occur in non-factive contexts, unlike fact-referring definite descriptions, they do not presume acquaintance with anything. (19a) does not say that I wonder/fear *de re* of a particular woman whether she is the one John is seeing. Rather, it says that I wonder/fear what the fact of the matter is concerning who John is seeing.

Recall that definite descriptions cannot stand in for free relatives occurring in specificational sentences that specify qualities of an individual, including location and height:

(21a) What the woman who John is seeing is is small.

(21b) The woman who John is seeing is small.

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(22a) Where the woman who John is seeing is is Scotland.

(22b) *The woman who John is seeing is Scotland.

Definite descriptions only refer to a particular fact. Let's say John is seeing Susan. When 'The woman who John is seeing' takes on its dual role, it both picks out a person, Susan, and characterises the description that truly holds of her, namely the fact that Susan is the woman who John is seeing. It thereby refers to the fact that Susan is the woman John is seeing. There is nothing in the content of the description that might permit reference to a fact concerning Susan's whereabouts or her size, so the infelicity of (24b), and the fact that the definite description in (23b) does not refer to a fact at all, is to be expected.

The evidence at the beginning of this section suggests that specificational sentences concern facts or propositions. I have shown that, and explained how, definite descriptions and free relatives can be used to refer to facts across different contexts. I will assume for the remainder of this paper that they refer to facts in specificational contexts.

5. FOP and other puzzles

I am now in a position to present my account of specificational sentences. The evidence presented in §3 suggests that the post-copular expression in specificational sentences is an elided declarative sentence. The evidence in §4 suggests that the NPs of specificational sentences refer to facts. Consider:

(35a) The thing I love is mountain climbing.

(35b) [The thing I love] is [I love mountain climbing.]

Suppose I love mountain climbing. 'The thing I love' takes on its dual role and refers to the fact that I love mountain climbing. This is precisely the fact that is the extension of the sentence occurring elliptically in post-copular position. A natural way to interpret (35a) is as expressing the identity of the fact referred to by the pre-copular clause and the fact denoted by the post-copular clause. We can write the truth-conditions informally as follows:

[[The thing I love is mountain climbing]] ∈ TRUE iff: the referent of

'The thing I love' = the extension of 'I love mountain climbing'.

Now consider a specificational sentence involving a free relative:

[[What I love is mountain climbing]] ∈ TRUE iff: the extension of 'I

love what' = the extension of 'I love mountain climbing'.

This gets the truth-conditions right. Both are true only in worlds where I love mountain climbing. Call this the 'fact-analysis' (FA). FA provides a somewhat unified account of specificational sentences, since the truth-conditions come out the same despite the different semantic and syntactic profiles of free relatives and definite descriptions. As demonstrated in the previous section, the differences attributed to free relatives and definite descriptions mean FA is compatible with the differences in their syntactic distribution. Moreover, FA does all this without positing a new kind of copula. FA interprets the 'is' in specificational sentences as the familiar 'is' of identity. FA suffers none of the drawbacks of QAA.

The claim that specificational sentences are kinds of equative sentences is not itself new. In fact, Philippe Schlenker's (2003) own brand of QAA bears some similarity to my own. It will be instructive to contrast Schlenkers account with FA

because Schlenkers analysis is subject to a problem that appears initially to plague FA. While Schlenker is forced to weaken the plausibility of his account, we shall see that FA elegantly avoids the problem.

Schlenker takes indirect questions to denote the proposition expressed by their own unique and exhaustive answer (as in Groenendijk and Stokhof 1997). He takes specificational sentences to equate the propositions denoted by the pre-copular clause with the proposition expressed by the post-copular clause. Suppose that the unique and exhaustive true answer to 'What does John love?' is 'John loves only himself'. On Schlenker's analysis 'What John loves' denotes <John loves only John>. 'What John loves is himself' is then understood as identifying this proposition with the proposition expressed by the post-copular clause, <John loves John>. The problem is that these propositions are not identical: <John loves John> can be true if John loves Mary as well. Implausibly, this means the sentence can never be true.

Schlenker's suggested solution weakens the plausibility of his analysis. He claims that the post-copular clause pragmatically implicates the stronger proposition denoted by the pre-copular clause before the two compose into the equation (2003: 24-27). Even if we assume 'John loves himself' implicates that John loves only himself, that an utterance usually carries an implicature does not establish that the relevant pragmatic process is pre-semantic. Schlenker is not alone in thinking that pre-semantic pragmatic processes are commonplace (he cites the work of Cherchia 2000, but see Recanati 2010 for a more comprehensive project). However, where possible, one should not resort to this. Semantics assigns meaning to sentences and develops rules showing how they are functions of the meanings of their parts parts; pragmatics concerns the meanings of linguistic acts, and their re-

lation to the independent deliverances of semantics. Giving a semantic-pragmatic account of the truth-conditions of sentences serves only to muddy this distinction and generates confusion about the purpose of providing truth-conditions for sentences of natural language.

Adopting FA yields a more elegant solution to this problem. I have not said much about what facts are because the metaphysics of facts is beyond the scope of this paper. Here, facts are just the referents of certain sentence nominalizations. However, the role I have given them in my theory does tell us something about them. Along with Kratzer, we might take them to be the parts of worlds that true propositions correspond to, such that, if a fact f corresponds to a proposition p, then f should only involve things that are relevant to p's truth.

Now lets suppose that John loves only himself and consider the propositions <John loves only John> and <John loves John>. What part of the world is most relevant to the truth of the former? The part that involves John's loving himself and no one else—the fact that John loves only himself. In this circumstance, it is the very same part of the world that is relevant to the truth of <John loves John>. In a world where John loves only himself, what difference could there be between the part where John loves himself and the part where John loves only himself? Despite the fact that they are different propositions, in worlds where they are both true, they plausibly correspond to the same fact. Now suppose that John loves Mary as well as himself. In this scenario, there is no part of the world that involves John's only loving himself, so the fact that John loves only himself doesnt obtain and <John loves only John> is false. In this scenario, <John loves John> corresponds to the part of the world that involves John's loving John—the fact that John loves John. Since this obtains, the proposition is true. In worlds where John loves more

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than himself, the corresponding facts differ, so 'What John loves is himself' comes out false. FA gets the truth-conditions right with no need for suspect linguistic apparatus. I am now in a position to give the truth-conditions of (1):

[[The number of planets is eight]] $\in$ TRUE iff: the referent of 'The number of planets' = the extension of 'There are eight planets'.

This analysis inherits the same explanation of FOP from the QAA analysis. In line with the evidence presented in §3, FA assumes that the post-copular expression of a specificational sentence is an elided declarative sentence. In this case, the sentence is 'There are eight planets', in which the cardinal 'eight' occurs in its typical syntactic position and contributes its typical semantic value in determining the extension of the sentence.

6.   Conclusions

The analysis of sentences such as (1) I have presented yields a solution to FOP. FA is not the only analysis to do this, but to my knowledge, it is the only analysis that does so while being compatible with all the linguistic evidence presented in this paper. The Fregean analysis neither yields a solution to FOP, nor explains the available linguistic evidence, so we may conclude that the Fregean analysis is mistaken. QAA is only compatible with the available evidence concerning the syntactic distribution of definite descriptions and free relatives if it is supplemented in an ad hoc manner. We may also conclude that QAA is mistaken. More tentatively, we may conclude that FA is on the right track. These conclusions are of great significance for the philosophy of language.

However, the issues I started with were ontological. Recall the easy argument: 'The following sentence is true only if numbers exist:

(1) The number of planets in the solar system is eight.

(1) is true; hence, numbers exist.' This argument is only sound if (1) implies the existence of numbers. We have seen that (1) does not contain any expressions that purport to refer numbers. The referring expression it does contain, 'The number of planets in the solar system', refers only to the fact that there are eight planets in the solar system. Therefore, as it stands, the above argument is unsound.

Is there any way the realist might salvage it? One way is to seek out other sentences that more obviously appear to be about numbers. 'The number seven is prime', for example. However, an argument involving this sentence would not have the intuitive appeal of the easy argument. Part of what makes the above argument seem so powerful is that 'The number of planets in the solar system is eight' appears undeniably true. Once pure mathematical sentences enter the scene, intuitions are not so strong, and familiar philosophical concerns about the subject matter of mathematics arise. Another way might be to argue, on independent metaphysical grounds, that mathematical objects must exist in order for facts such as the one mentioned by (1) to obtain. This would render the appeal to natural language pointless. If numbers must exist for there to be eight planets, then the meaning of 'The number of planets in the solar system is eight' is neither here nor there; what matters is that there are eight planets. The lesson to be drawn in light of the failure of the easy argument is that establishing the existence of mathematical objects is not easy.

Acknowledgements

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